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Fundamentals of soft category theory

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Abstract

The soft category theory offers a way to study soft theories developed so far more generally. The main purpose of this paper is to introduce the basic algebraic operations in soft categories, and for that we introduce some algebraic operations, like intersection and union, in categories. Also, the notion of composition of soft functors is introduced to form category of all soft categories.

Keywords: Category theory, soft set, soft category, algebraic operations in soft category, algebraic operations on category, composition of soft functors.

1 Introduction

Molodtsov [1] introduced the concept of soft sets to overcome the difficulties that arise while dealing with complicated problems involving uncertainties in economics, engineering, environmental science, medical science and social science where neither methods of classical mathematics nor mathematical theories such as probability theory, fuzzy set theory, rough set theory, vague set theory and the interval mathematics can be successfully used. In soft set theory, the problem of setting the membership function does not arise, which makes the theory easily applicable to many different fields, see [2–6]. At present, works on soft theories are progressing rapidly. The algebraic

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structure of soft sets has been studied by some authors, for example see [7–14]. Maji et al. [15] introduced several operations on soft sets. Aktaş and Çağman [16] defined soft groups and obtained the main properties of these groups. They also compared soft sets with fuzzy sets and rough sets. Besides, Jun [17] defined soft ideals on BCK/BCI-algebras. Feng et al. [18] defined soft semirings, soft ideals on soft semirings and idealistic soft semirings, also see [19]. Yamak et al. [20] introduced tjhe notion of soft hyperstructures. Acar et al. [21] defined soft rings. Qiu-Mei Sun et al. [22] defined the concept of soft modules and studied their basic properties. Sardar and Gupta [23] introduced the notions of soft category and soft functor and studied properties of them in details. The present paper is a sequel to this.

The main purpose of this paper is to introduce basic algebraic operations on soft categories, for which we firstly define those operations on categories. We observe that most of the operations on soft sets defined in [15] and [24] are particular cases of the operations on soft category defined by us. Also, the notion of composition of soft functors is introduced to form category of all soft categories.

2 Preliminaries

We assume that reader is familiar to the notations of category theory [25–31]. In this section, we recall some basic definitions of soft set theory and soft category theory.

Definition 1. *[1] Let U be an initial universe set, E be a set of parameters, $P(U)$ be the power set of U, and $A \subseteq E$. A pair(F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$.*

In other words, a soft set over U is a parameterized family of subsets of the universe U. To illustrate this idea, let us consider the following example.

> Let us consider a soft set (F, E) which describes the attractiveness of houses that Mr.X is considering for purchase. Suppose that there are six houses in the universe $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ under consideration, and that $E = \{e_1, e_2, e_3, e_4, e_5\}$ is a set of decision parameters. Let e_1 = expensive, e_2 = beautiful, e_3 = wooden, e_4 = cheap, and e_5 = in green surroundings. In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on.

Now, we recall the following definitions from [15, 24].

- Let (F, A) be soft set over U. Then, (F, A) is called a *soft null set* if $F(x) = \emptyset$ for all $x \in A$.

- Let (F, A) and (G, B) be soft sets over a common universe U. Then, (G, B) is called a *soft subset* of (F, A), denoted by $(F, A)\tilde{\subset}(G, B)$, if it satisfies the followings:
  (1) $B \subseteq A$;
  (2) For all $x \in B$, $F(x)$ and $G(x)$ are identical approximations.

- Let (F, A) and (G, B) be two soft sets over U. Then, they are said to be *equal* if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

- Let (F, A) and (G, B) be soft sets over a common universe U. Then, "(F, A) AND (G, B)", denoted by $(F, A)\tilde{\wedge}(G, B)$, is defined by

  $$(F, A)\tilde{\wedge}(G, B) = (H, A \times B),$$

  where $H(x, y) = F(x) \cap G(y)$ for all $(x, y) \in A \times B$.

- Let (F, A) and (G, B) be soft sets over a common universe U. Then, "(F, A) OR (G, B)", denoted by $(F, A)\tilde{\vee}(G, B)$, is defined by

  $$(F, A)\tilde{\vee}(G, B) = (H, A \times B),$$

  where $H(x, y) = F(x) \cup G(y)$ for all $(x, y) \in A \times B$.

- Let (F, A) and (G, B) be soft sets over a common universe U. Then, the *union* of (F, A) and (G, B), denoted by $(F, A)\tilde{\cup}(G, B)$, is defined by $(F, A)\tilde{\cup}(G, B) = (H, C)$, where $C = A \cup B$ and for all $e \in C$,

  $$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cup G(e) & \text{if } e \in A \cap B. \end{cases}$$

- Let (F, A) and (G, B) be soft sets over a common universe U such that $A \cap B \neq \emptyset$. Then, the *restricted union* of (F, A) and (G, B), denoted by $(F, A) \cup_R (G, B)$, is defined by $(F, A) \cup_R (G, B) = (H, C)$, where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) \cup G(e)$.

- Let (F, A) and (G, B) be soft sets over a common universe U such that $A \cap B \neq \emptyset$. Then, the *intersection* of (F, A) and (G, B), denoted by $(F, A)\tilde{\cap}(G, B)$, is defined by $(F, A)\tilde{\cap}(G, B) = (H, C)$, where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) or G(e)$ (as both are same set).

In [24], it had been pointed out that this definition of intersection is not well-defined, which was explained with the following example.

> [24] Consider two soft sets (F, A) and (G, B), where the universe U is a set of houses; $U = \{h1, h2, h3, h4, h5, h6\}$, and $A = \{wooden, beautiful\}$, and $B = \{beautiful\}$. Let $F(wooden) = \{h_1, h_3\}$, $F(beautiful) = \{h_2, h_4\}$, $G(beautiful) = \{h_4\}$. Now, consider $(F, A)\tilde{\cap}(G, B) = (H, C)$. Since "$beautiful'' \in A \cap B$, we have $H(beautiful) = F(beautiful) = \{h_2, h_4\} \neq \{h_4\} = G(beautiful) = H(beautiful)$, and this is a contradiction.

Therefore, the intersection is now defined in the following way, which is also known as "restricted" intersection [24].

- Let (F, A) and (G, B) be soft sets over a common universe U such that $A \cap B \neq \emptyset$. Then, the *restricted intersection* of (F, A) and (G, B), denoted by $(F, A) \cap_R (G, B)$, is defined by $(F, A) \cap_R (G, B) = (H, C)$, where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) \cap G(e)$.
- Let (F, A) and (G, B) be soft sets over a common universe U. Then, the *extended intersection* of (F, A) and (G, B), denoted by $(F, A) \cap_E (G, B)$, is defined by $(F, A) \cap_E (G, B) = (H, C)$, where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cap G(e) & \text{if } e \in A \cap B. \end{cases}$$

Now we recall some definitions of soft category .

Definition 2. *[23] Let C be a category, $P(C)$ be the set of all subcategories of C and A be a set of parameters. Let $F : A \to P(C)$ be a mapping. Then, (F, A) is said to be a soft category over C if $F(x)$ is a subcategory of C, i.e., it is nothing but a parameterized family of subcategories of a category.*

[23] Let SET be the category of all sets where the arrows are the set mappings and $A = N =$ Set of all natural numbers. Also, let $F(n)$ be the subcategory of the category SET consisting of all sets having cardinality n, for all $n \in N$. Hence, (F, A) is a soft category over the category SET.

[23] Let GRP be the category of all groups, where the arrows are the group homomorphisms. Also, let $A = \{cyclic, finite, commutative, free\}$. Then, (F, A) is a soft category over GRP, where $F(x)$ is the subcategory of all groups with the property x. Hence, it is nothing but to point out cyclic groups or finite groups etc.

Definition 3. *[23] Let (F, A) and (H, B) be two soft categories over C. Then, we say that, (H, B) is a soft subcategory of (F, A) if the followings are satisfied:*

(1) $B \subseteq A$,

(2) $H(x)$ is a subcategory of $F(x)$, for all $x \in B$.

[23] Let (F, A) be the soft category of example 2 and (H, B) be another soft category over GRP, where $B = \{cyclic\}$ and $H(cyclic)$ be the subcategory of all finite cyclic groups. Then, clearly (H, B) is a soft subcategory of (F, A).

Definition 4. *[23] Two soft categories (F, A) and (H, B) over same category C is said to be equal if (H, B) is a soft subcategory of (F, A) and (F, A) is a soft subcategory of (H, B).*

Definition 5. *[23] Let (F, A) be a soft category over C and C^{op} be the dual category of C. Then, $(F, A)^{op} = (F^{op}, A)$ is said to be the dual soft category of (F, A) if $F^{op}(x)$ corresponds to the dual subcategory of $F(x)$, for all $x \in A$. Clearly $(F, A)^{op}$ is a soft category over C^{op}.*

Definition 6. *Let (F, A) be a soft category over C and P be a certain property of categories. Then, we say that (F, A) is a soft category with property P, if for all $x \in A$, $F(x)$ as a category has the property P.*

In the above definition P may be any property of a category. In [23], we defined full soft category, balanced soft category, normal soft category, soft category with limits and many more like these. Here in the above definition what we try to mean is if we take P as “full” or say “balanced”, then the above definition yields the definition of full soft category or balanced soft category as they are defined in [23]

Definition 7. *[23] Let (F, A) over C and (H, B) over D be two soft categories. Also, suppose that $g : A \to B$ is a set mapping and $K : C \to D$ is a functor [30]. Then, (K, g) is said to be a soft functor from (F, A) to (H, B) if*

(1) K is full [30], i.e., image of C under K is all of D,

(2) g is a mapping from A onto B, and

(3) $K(F(x)) = H(g(x))$ for all $x \in A$.

3 Algebraic operations in categories

This section contains the introduction of intersection and union of categories and some of their properties.

Definition 8. *Let C and D be two categories. Then, the intersection of two categories C and D will be denoted by $C \cap D$, and defined to be as follows:*

(1) $Ob(C \cap D) = Ob(C) \cap Ob(D)$,

(2) $Hom_{C \cap D}[A, B] = Hom_C[A, B] \cap Hom_D[A, B]$ for all $A, B \in Ob(C \cap D)$.

According to this definition, it can be easily verified that $C \cap D$ is again a category. Also, we see that $C \cap D$ and $D \cap C$ are the same category. Moreover. we can induce this definition for intersection of a family of categories.

Definition 9. *Let C and D be two categories. Then, the union of two categories C and D will be denoted by $C \cup D$, and defined to be as follows:*

(1) $Ob(C \cup D) = Ob(C) \cup Ob(D)$,

(2) $Hom_{C \cup D}[A, B] = Hom_C[A, B] \cup Hom_D[A, B]$ for all $A, B \in Ob(C \cup D)$.

But this union $C \cup D$ is not necessarily a category. We illustrate this in the following example.

Let us consider two categories E and D, where

$$\begin{aligned} &Ob(E) = \{A, B\}, \quad Hom[A, A] = \{I_A\}, \quad Hom[B, B] = \{I_B\}, \\ &Hom[A, B] = \{f\}, \quad Hom[B, A] = \emptyset \end{aligned}$$

and

$$\begin{array}{lll} Ob(D) = \{B, C\}, & Hom[C, C] = \{I_C\}, & Hom[B, B] = \{I_B\}, \\ Hom[B, C] = \{g\}, & Hom[C, B] = \emptyset. & \end{array}$$

Then, by the previous definition, $Ob(E \cup D) = Ob(E) \cup Ob(D) = \{A, B, C\}$ and

$$\begin{array}{lll} Hom[A, A] = \{I_A\}, & Hom[B, B] = \{I_B\}, & Hom[C, C] = \{I_C\}, \\ Hom[A, B] = \{f\}, & Hom[B, A] = \emptyset, & Hom[B, C] = \{g\}, \\ Hom[C, B] = \emptyset, & Hom[A, C] = \emptyset, & Hom[C, A] = \emptyset. \end{array}$$

Now as $f \in Hom[A, B]$ and $g \in Hom[B, C]$, but $f \circ g \in Hom[A, C] = \emptyset$ is a contradiction.

Though we find that, according to the previous definition, union of two categories is not necessarily a category, but we also observe that there is a smallest category containing the union $E \cup D$. Here that category, say E, is $Ob(M) = Ob(E) \cup Ob(D) = \{A, B, C\}$ and $Hom[A, A] = \{I_A\}, Hom[B, B] = \{I_B\}, Hom[C, C] = \{I_C\}, Hom[A, B] = \{f\}, Hom[B, A] = \emptyset, Hom[B, C] = \{g\}, Hom[C, B] = \emptyset, Hom[A, C] = f \circ g, Hom[C, A] = \emptyset$. Thus, we get the following definition.

Definition 10. *Let C and D be two categories. Then, the category generated by $C \cup D$ is denoted by $C \tilde{\cup} D$ and is defined to be the smallest category containing both C and D as subcategories, i.e., the intersection of all categories containing both C and D as subcategories. We see that, the category $C \tilde{\cup} D$ contains the arrows of the following forms:*

(1) arrows of the category C,

(2) arrows of the category D,

(3) arrows of the form $f \circ g$ where f is an arrow of C and g is an arrow of D,

(4) arrows of the form $g \circ f$ where f is an arrow of C and g is an arrow of D.

The following is easily derivable from the above definitions.

Theorem 1. *$Ob(C\tilde{\cup}D) = Ob(C \cup D)$. Moreover, if $Ob(C \cap D) = \emptyset$, then $C\tilde{\cup}D = C \cup D$.*

Theorem 2. *If C, D and E are three categories, then*

(1) $C \cap (D \cap E) = (C \cap D) \cap E$.

(2) $C\tilde{\cup}(D\tilde{\cup}E) = (C\tilde{\cup}D)\tilde{\cup}E$.

Proof. (1) We have

$$\begin{aligned} Ob(C \cap (D \cap E)) &= Ob(C) \cap Ob(D \cap E) \\ &= Ob(C) \cap (Ob(D) \cap Ob(E)) \\ &= (Ob(C) \cap Ob(D)) \cap Ob(E)) \\ &= Ob(C \cap D) \cap Ob(E) \\ &= Ob((C \cap D) \cap E). \end{aligned}$$

In the similar way, we can show that, for any $A, B \in Ob(C \cap (D \cap E))$, $Hom[A, B]$ in both the categories are equal. Hence, the proof is completed.

(2) According to the definition, both the categories

$$C\tilde{\cup}(D\tilde{\cup}E) \text{ and } (C\tilde{\cup}D)\tilde{\cup}E$$

refer to the same category, which is the smallest category containing C, D and E. Hence, we get the result. □

Theorem 3. *If C and D are two categories, then*

(1) $(C \cap D)^{op} = C^{op} \cap D^{op}$.

(2) $(C\tilde{\cup}D)^{op} = C^{op}\tilde{\cup}D^{op}$.

Proof. The equality of objects is too trivial to show. So, we show here the equality of arrows only.

(1) We have

$$\begin{aligned} Hom_{(C\cap D)^{op}}[A, B] &= Hom_{(C\cap D)}[B, A] \\ &= Hom_C[B, A] \cap Hom_D[B, A] \\ &= Hom_{C^{op}}[A, B] \cap Hom_{D^{op}}[A, B] \\ &= Hom_{C^{op}\cap D^{op}}[A, B], \end{aligned}$$

for each object A and B. Therefore, the proof is completed.

(2) Suppose that $A, B \in Ob((C\tilde{\cup}D)^{op})$ and $f \in Hom_{(C\tilde{\cup}D)^{op}}[A, B]$. Then, $f \in Hom_{(C\tilde{\cup}D)}[B, A]$. So, by definition, f is of following forms:

(a) arrow of the category C,

(b) arrow of the category D,

(c) arrow of the form $h \circ g$ where h is an arrow of C and g is an arrow of D,

(d) arrow of the form $g \circ h$ where h is an arrow of C and g is an arrow of D.

In the cases (a) and (b), clearly $f \in Hom_{C^{op}\tilde{\cup}D^{op}}[A, B]$. For the case (c), as h and g belongs to C^{op} and D^{op}, respectively, just altering their directions, so direction of f is also altered and it becomes $g \circ h$ in $C^{op}\tilde{\cup}D^{op}$. The case (d) is same as (c).

Conversely, suppose that $A, B \in Ob(C^{op}\tilde{\cup}D^{op})$ and $f \in Hom_{C^{op}\tilde{\cup}D^{op}}[A, B]$. Then, is of following forms:

(a) arrow of the category C^{op},

(b) arrow of the category D^{op},

(c) arrow of the form $h \circ g$, where h is an arrow of C^{op} and g is an arrow of D^{op},

(d) arrow of the form $g \circ h$ where h is an arrow of C^{op} and g is an arrow of D^{op}.

In the cases (a) and (b), clearly $f \in Hom_{(C\tilde{\cup}D)^{op}}[A, B]$. For case (c), $g \circ h$ is in the category $(C\tilde{\cup}D)$ and so $f = h \circ g$ is in $(C\tilde{\cup}D)^{op}$. The case (d) is same as (c).
Therefore, the two categories are equal. ☐

Theorem 4. *If C, D and E are three categories, then $C \times (D \cap E) = (C \times D) \cap (C \times E)$.*

Proof. We have

$$\begin{aligned} Ob(C \times (D \cap E)) &= Ob(C) \times Ob(D \cap E) \\ &= Ob(C) \times (Ob(D) \cap Ob(E)) \\ &= (Ob(C) \times Ob(D) \cap (Ob(C) \times Ob(E)) \\ &= Ob((C \times D) \cap (C \times E)). \end{aligned}$$

The equality of arrows can be shown similarly. ☐

Theorem 5. *If C, D and E are three categories, then $C \times (D \tilde{\cup} E) = (C \times D) \tilde{\cup} (C \times E)$.*

Proof. The equality of objects can be shown using Theorem 1 and following the same technique as we adopted in the previous theorem. Now, let us consider an arrow (f, g) of $C \times (D \tilde{\cup} E)$. Then, f is an arrow of C and g is an arrow of $D \tilde{\cup} E$. So, g is of following forms:

(a) arrow of the category D,

(b) arrow of the category E,

(c) arrow of the form $h \circ k$ where h is an arrow of D and k is an arrow of E,

(d) arrow of the form $k \circ h$ where h is an arrow of D and k is an arrow of E.

For cases (a) and (b), (f, g) becomes an arrow of $(C \times D) \tilde{\cup} (C \times E)$. For case (c), we observe that, $(f, g) = (f, h) \circ (i, k)$, where i is an identity arrow of C so that the composition is defined. Hence, (f, g) becomes an arrow of $(C \times D) \tilde{\cup} (C \times E)$. The case (d) is same as (c).

Conversely, consider an arrow k of $(C \times D) \tilde{\cup} (C \times E)$. Then, k is of the following forms:

(a) arrow of the category $C \times D$,

(b) arrow of the category $C \times E$,

(c) arrow of the form $(h_1 \times h_2) \circ (g_1 \times g_2)$ where $(h_1 \times h_2)$ is an arrow of $C \times D$ and $(g_1 \times g_2)$ is an arrow of $C \times E$,

(d) arrow of the form $(g_1 \times g_2) \circ (h_1 \times h_2)$ where $(h_1 \times h_2)$ is an arrow of $C \times D$ and $(g_1 \times g_2)$ is an arrow of $C \times E$.

In the cases (a) and (b), clearly k becomes an arrow of $C \times (D \tilde{\cup} E)$. For the case (c), $k = (h_1 \circ g_1, h_2 \circ g_2)$. As h_2 and g_2 are in D and E respectively, so $h_2 \circ g_2$ becomes an arrow of $D \tilde{\cup} E$ and hence k becomes an arrow of $C \times (D \tilde{\cup} E)$. The case (d) is same as (c). Therefore, we get the required equality. □

Theorem 6. *If C, D and E are three categories, then $C\tilde{\cup}(D\cap E)$ is a full subcategory of $(C\tilde{\cup}D)\cap(C\tilde{\cup}E)$.*

Proof. We first observe that

$$\begin{aligned} Ob(C\tilde{\cup}(D\cap E)) &= Ob(C\cup(D\cap E)) \\ &= Ob((C\cap D)\cup(C\cap E)) \\ &= Ob((C\cap D)\tilde{\cup}(C\cap E)). \end{aligned}$$

Now, let us consider an arrow of h in $C\tilde{\cup}(D\cap E)$. Then, by definition, the following cases are to be considered:

Case 1. If h is an arrow of C, then it is an arrow of both $C\tilde{\cup}D$ and $C\tilde{\cup}E$. So h is an arrow of $(C\tilde{\cup}D)\cap(C\tilde{\cup}E)$.

Case 2. If h is an arrow of $D\cap E$, then also it is an arrow of both $C\tilde{\cup}D$ and $C\tilde{\cup}E$. So h is an arrow of $(C\tilde{\cup}D)\cap(C\tilde{\cup}E)$.

Case 3. If h is neither an arrow of C nor an arrow of $D\cap E$, then there are arrows f in C and g in $D\cap E$ such that $h=f\circ g$ or $h=g\circ f$. In both cases this composition becomes arrows of both $C\tilde{\cup}D$ and $C\tilde{\cup}E$. Hence, h is an arrow of $(C\tilde{\cup}D)\cap(C\tilde{\cup}E)$.

Therefore, the proof is completed. □

The following example shows that the equality does not hold always in the above theorem.

Let us consider $\mathbb{Z}$ and $\mathbb{N}$ as the set of integers and the set of non-negative integers, respectively. We define $f:\mathbb{Z}\to\mathbb{N}$ as $f(x)=x^2$ and $g:\mathbb{Z}\to\mathbb{N}$ as $g(x)=|x|$. Let $A=\{-1,0,1\}$ be a set and h be the inclusion mapping from A to $\mathbb{Z}$. Then, clearly the composition mappings $f\circ h$ and $g\circ h$ are equal. Now, we construct three categories C, D and E as follows:

(1) $Ob(C)=\{A,\mathbb{Z}\}$, and $Hom[A,A]=\{i_A\}$, $Hom[\mathbb{Z},\mathbb{Z}]=\{i_{\mathbb{Z}}\}$, $Hom[A,\mathbb{Z}]=\{h\}$, $Hom[\mathbb{Z},A]=\emptyset$;

(2) $Ob(D)=\{\mathbb{N},\mathbb{Z}\}$, and $Hom[\mathbb{N},\mathbb{N}]=\{i_{\mathbb{N}}\}$, $Hom[\mathbb{Z},\mathbb{Z}]=\{i_{\mathbb{Z}}\}$, $Hom[\mathbb{Z},\mathbb{N}]=\{f\}$, $Hom[\mathbb{N},\mathbb{Z}]=\emptyset$;

(3) $Ob(E)=\{\mathbb{N},\mathbb{Z}\}$, and $Hom[\mathbb{N},\mathbb{N}]=\{i_{\mathbb{N}}\}$, $Hom[\mathbb{Z},\mathbb{Z}]=\{i_{\mathbb{Z}}\}$, $Hom[\mathbb{Z},\mathbb{N}]=\{g\}$, $Hom[\mathbb{N},\mathbb{Z}]=\emptyset$.

In the above, we denote the identity mapping on a set X as i_X. Now, we see that the composition arrow $f\circ h=g\circ h$ becomes an arrow of

$(C\tilde{\cup}D)\cap(C\tilde{\cup}E)$ but this arrow does not belongs to the category $C\tilde{\cup}(D\cap E)$. Hence, we showed that the equality in the above theorem does not hold always.

Theorem 7. *If C, D and E are three categories, then $(C\cap D)\tilde{\cup}(C\cap E)$ is a full subcategory of $C\cap(D\tilde{\cup}E)$.*

Proof. We first observe that

$$\begin{aligned} Ob(C\cap(D\tilde{\cup}E)) &= Ob(C\cap(D\cup E)) \\ &= Ob((C\cup D)\cap(C\cup E)) \\ &= Ob((C\tilde{\cup}D)\cap\tilde{\cup}(C\tilde{\cup}E)). \end{aligned}$$

Now, let us consider an arrow of h in $(C\cap D)\tilde{\cup}(C\cap E)$. Then, by the definition, the following cases are to be considered:

Case 1. If h is an arrow of $(C\cap D)$, then it is an arrow of both C and $D\tilde{\cup}E$. So h is an arrow of $C\cap(D\tilde{\cup}E)$.

Case 2. If h is an arrow of $(C\cap E)$, then it is an arrow of both C and $D\tilde{\cup}E$. So h is an arrow of $C\cap(D\tilde{\cup}E)$.

Case 3. If h is neither an arrow of $C\cap D$ nor an arrow of $C\cap E$, then there are arrows f in $C\cap D$ and g in $C\cap E$ such that $h=f\circ g$ or $h=g\circ f$. In both cases the composition becomes arrows of both C and $D\tilde{\cup}E$. Hence h is an arrow of $C\cap(D\tilde{\cup}E)$.

Therefore, the proof is completed. ☐

The following example shows that the equality does not hold always in the above theorem.

First we consider A, $\mathbb{Z}$, $\mathbb{N}$, f, h and $f\circ h$ as in the previous example. Now, we construct three categories C, D and E as follows:

1. $Ob(C)=\{A,\mathbb{N}\}$, and $Hom[A,A]=\{i_A\}$, $Hom[\mathbb{N},\mathbb{N}]=\{i_{\mathbb{N}}\}$, $Hom[A,\mathbb{N}]=\{f\circ h\}$, $Hom[\mathbb{N},A]=\emptyset$;
2. $Ob(D)=\{A,\mathbb{Z}\}$, and $Hom[A,A]=\{i_A\}$, $Hom[\mathbb{Z},\mathbb{Z}]=\{i_{\mathbb{Z}}\}$, $Hom[A,\mathbb{Z}]=\{h\}$, $Hom[\mathbb{Z},A]=\emptyset$;
3. $Ob(E)=\{\mathbb{N},\mathbb{Z}\}$, and $Hom[\mathbb{N},\mathbb{N}]=\{i_{\mathbb{N}}\}$, $Hom[\mathbb{Z},\mathbb{Z}]=\{i_{\mathbb{Z}}\}$, $Hom[\mathbb{Z},\mathbb{N}]=\{f\}$, $Hom[\mathbb{N},\mathbb{Z}]=\emptyset$.

In the above, we denote the identity mapping on a set X as i_X. Now, we see that the arrow $f\circ h$ becomes an arrow of $C\cap(D\tilde{\cup}E)$ but this arrow does not belongs to the category $(C\cap D)\tilde{\cup}(C\cap E)$. Hence, we conclude that the equality in the above theorem does not hold always.

4 Algebraic operations in soft categories

In this section, we introduce the notion of AND, OR, intersection, union and product of two soft categories. Also, we present some results involving them.

Definition 11. *Let* (F, A) *over* C *and* (G, B) *over* D *be two soft categories. Then, "*(F, A) *AND* (G, B)*", denoted by* $(F, A)\tilde{\wedge}(G, B)$*, is defined by* $(F, A)\tilde{\wedge}(G, B) = (H, A \times B)$ *where* $H(x, y) = F(x) \cap G(y)$ *for all* $(x, y) \in A \times B$*.*

We see that, $(F, A)\tilde{\wedge}(G, B)$ is again a soft category over $C\tilde{\cup}D$.

Definition 12. *Let* (F, A) *over* C *and* (G, B) *over* D *be two soft categories. Then, "*(F, A) *OR* (G, B)*", denoted by* $(F, A)\tilde{\vee}(G, B)$*, is defined by* $(F, A)\tilde{\vee}(G, B) = (H, A \times B)$ *where* $H(x, y) = F(x)\tilde{\cup}G(y)$ *for all* $(x, y) \in A \times B$*.*

We see that, $(F, A)\tilde{\vee}(G, B)$ is also a soft category over $C\tilde{\cup}D$.

Definition 13. *Let* (F, A) *over* C *and* (G, B) *over* D *be two soft categories such that* $A \cap B \neq \emptyset$*. Then,the intersection of these two soft categories,denoted by* $(F, A) \cap (G, B)$*, is defined by* $(F, A) \cap (G, B) = (H, A \cap B)$ *where* $H(e) = F(e) \cap G(e)$ *for all* $e \in A \cap B$*.*

Definition 14. *Let* (F, A) *over* C *and* (G, B) *over* D *be two soft categories. Then, the extended intersection of these two soft categories,denoted by* $(F, A) \cap_E (G, B)$*, is defined by* $(F, A) \cap_E (G, B) = (H, A \cup B)$*, where*

$$H(e) = \begin{cases} F(e) & \textit{if } e \in A - B \\ G(e) & \textit{if } e \in B - A \\ F(e) \cap G(e) & \textit{if } e \in A \cap B. \end{cases}$$

Definition 15. *Let* (F, A) *over* C *and* (G, B) *over* D *be two soft categories. Then, the union of these two soft categories, denoted by* $(F, A)\tilde{\cup}(G, B)$*, is defined by* $(F, A)\tilde{\cup}(G, B) = (H, A \cup B)$*, where*

$$H(e) = \begin{cases} F(e) & \textit{if } e \in A - B \\ G(e) & \textit{if } e \in B - A \\ F(e)\tilde{\cup}G(e) & \textit{if } e \in A \cap B. \end{cases}$$

Definition 16. *Let* (F, A) *over* C *and* (G, B) *over* D *be two soft categories such that* $A \cap B \neq \emptyset$. *Then, the restricted union of these two soft categories,denoted by* $(F, A)\tilde{\cup}_R(G, B)$, *is defined by* $(F, A)\tilde{\cup}_R(G, B) = (H, A \cap B)$ *where* $H(e) = F(e)\tilde{\cup}G(e)$ *for all* $e \in A \cap B$.

We observe that intersection, extended intersection, union, restricted union, defined above, are soft categories over $C\tilde{\cup}D$.

Definition 17. *Let* (F, A) *over* C *and* (G, B) *over* D *be two soft categories. Then, the product of these two soft categories, denoted by* $(F, A) \times (G, B)$, *is defined by* $(F, A) \times (G, B) = (H, A \times B)$ *where* $H(x, y) = F(x) \times G(y)$ *for all* $(x, y) \in A \times B$.

Eventually this product of soft categories becomes a soft category over $C \times D$.

Now, we observe some properties of these operations.

Throughout this part of this section, we consider (F_1, A_1), (F_2, A_2), (F_3, A_3) are soft categories over C, D and E.

Theorem 8. *We have*

$$(F_1, A_1) \cap ((F_2, A_2) \cap (F_3, A_3)) = ((F_1, A_1) \cap (F_2, A_2)) \cap (F_3, A_3).$$

Proof. Indeed, we have

$$\begin{aligned}
&(F_1, A_1) \cap ((F_2, A_2) \cap (F_3, A_3))\\
&= (F_1, A_1) \cap (F_4, A_2 \cap A_3),\\
&\qquad \text{where } F_4(e) = F_2(e) \cap F_3(e), \text{ for } e \in A_2 \cap A_3\\
&= (F_5, A_1 \cap (A_2 \cap A_3)),\\
&\qquad \text{where } F_5(e) = F_1(e) \cap (F_2(e) \cap F_3(e)), \text{ for } e \in A_1 \cap (A_2 \cap A_3)\\
&\qquad \text{Applying Theorem 2 we get,}\\
&= (F_5, (A_1 \cap A_2) \cap A_3),\\
&= (F_6, A_1 \cap A_2) \cap (F_3, A_3),\\
&\qquad \text{where } F_6(e) = F_1(e) \cap F_2(e), \text{ for } e \in A_1 \cap A_2\\
&= ((F_1, A_1) \cap (F_2, A_2)) \cap (F_3, A_3).
\end{aligned}$$

□

Theorem 9. *We have*

$$(F_1, A_1) \cap_E ((F_2, A_2) \cap_E (F_3, A_3)) = ((F_1, A_1) \cap_E (F_2, A_2)) \cap_E (F_3, A_3).$$

Proof. The proof is similar to the proof of Theorem 8. $\square$

Theorem 10. *We have*

$$(F_1,A_1)\tilde{\cup}_R((F_2,A_2)\tilde{\cup}_R(F_3,A_3)) = ((F_1,A_1)\tilde{\cup}_R(F_2,A_2))\tilde{\cup}_R(F_3,A_3).$$

Proof. Indeed, we have

$$\begin{aligned}
&(F_1,A_1)\tilde{\cup}_R((F_2,A_2)\tilde{\cup}_R(F_3,A_3))\\
&= (F_1,A_1)\tilde{\cup}_R(F_4,A_2\cap A_3),\\
&\qquad \text{where } F_4(e)=F_2(e)\tilde{\cup}F_3(e), \text{ for } e\in A_2\cap A_3\\
&= (F_5,A_1\cap(A_2\cap A_3)),\\
&\qquad \text{where } F_5(e)=F_1(e)\tilde{\cup}(F_2(e)\tilde{\cup}F_3(e)), \text{ for } e\in A_1\cap(A_2\cap A_3)\\
&\qquad \text{Applying Theorem 2 we get,}\\
&= (F_5,(A_1\cap A_2)\cap A_3),\\
&= (F_6,A_1\cap A_2)\tilde{\cup}_R(F_3,A_3),\\
&\qquad \text{where } F_6(e)=F_1(e)\tilde{\cup}F_2(e), \text{ for } e\in A_1\cap A_2\\
&= ((F_1,A_1)\tilde{\cup}_R(F_2,A_2))\tilde{\cup}_R(F_3,A_3).
\end{aligned}$$

$\square$

Theorem 11. *We have*

$$(F_1,A_1)\tilde{\cup}((F_2,A_2)\tilde{\cup}(F_3,A_3)) = ((F_1,A_1)\tilde{\cup}(F_2,A_2))\tilde{\cup}(F_3,A_3).$$

Proof. The proof is similar to the proof of Theorem 10. $\square$

Theorem 12. *We have*

$$(F_1,A_1)\times((F_2,A_2)\cap(F_3,A_3)) = ((F_1,A_1)\times(F_2,A_2))\cap((F_1,A_1)\times(F_3,A_3)).$$

Proof. Indeed, we have

$$\begin{aligned}
&(F_1,A_1)\times((F_2,A_2)\cap(F_3,A_3))\\
&= (F_1,A_1)\times(F_4,A_2\cap A_3),\\
&\qquad \text{where } F_4(e)=F_2(e)\cap F_3(e), \text{ for } e\in A_2\cap A_3\\
&= (F_5,A_1\times(A_2\cap A_3)),\\
&\qquad \text{where } F_5((e,h))=F_1(e)\times(F_2(h)\cap F_3(h)), \text{ for } (e,h)\in A_1\times(A_2\cap A_3)\\
&\qquad \text{Applying Theorem 4 we get,}\\
&= (F_5,(A_1\times A_2)\cap(A_1\times A_3)),\\
&= (F_6,A_1\times A_2)\cap(F_7,A_1\times A_3),\\
&\qquad \text{where } F_6((e,h))=F_1(e)\times F_2(h), \text{ for } (e,h)\in A_1\times A_2\\
&\qquad \text{and } \quad F_7((e,h))=F_1(e)\times F_3(h), \text{ for } (e,h)\in A_1\times A_3\\
&= ((F_1,A_1)\times(F_2,A_2))\cap((F_1,A_1)\times(F_3,A_3)).
\end{aligned}$$

$\square$

Theorem 13. *We have*

$$(F_1, A_1)\times((F_2, A_2)\cap_E(F_3, A_3)) = ((F_1, A_1)\times(F_2, A_2))\cap_E((F_1, A_1)\times(F_3, A_3)).$$

Proof. The proof is similar to the proof of Theorem 12. □

Theorem 14. *We have*

$$(F_1, A_1)\times((F_2, A_2)\tilde{\cup}_R(F_3, A_3)) = ((F_1, A_1)\times(F_2, A_2))\tilde{\cup}_R((F_1, A_1)\times(F_3, A_3)).$$

Proof. Indeed, we have

$$\begin{aligned}
&(F_1, A_1) \times ((F_2, A_2)\tilde{\cup}_R(F_3, A_3))\\
&= (F_1, A_1) \times (F_4, A_2 \cap A_3),\\
&\qquad \text{where } F_4(e) = F_2(e)\tilde{\cup}F_3(e), \ \text{for } e \in A_2 \cap A_3\\
&= (F_5, A_1 \times (A_2 \cap A_3)),\\
&\qquad \text{where } F_5((e,h)) = F_1(e) \times (F_2(h)\tilde{\cup}F_3(h)), \text{ for } (e,h) \in A_1 \times (A_2 \cap A_3)\\
&\qquad \text{Applying Theorem 5 we get,}\\
&= (F_5, (A_1 \times A_2) \cap (A_1 \times A_3)),\\
&= (F_6, A_1 \times A_2)\tilde{\cup}_R(F_7, A_1 \times A_3),\\
&\qquad \text{where } F_6((e,h)) = F_1(e) \times F_2(h), \ \text{for } (e,h) \in A_1 \times A_2\\
&\qquad \text{and } \ F_7((e,h)) = F_1(e) \times F_3(h), \ \text{for } (e,h) \in A_1 \times A_3\\
&= ((F_1, A_1) \times (F_2, A_2))\tilde{\cup}_R((F_1, A_1) \times (F_3, A_3)).
\end{aligned}$$

□

Theorem 15. *We have*

$$(F_1, A_1) \times ((F_2, A_2)\tilde{\cup}(F_3, A_3)) = ((F_1, A_1) \times (F_2, A_2))\tilde{\cup}((F_1, A_1) \times (F_3, A_3)).$$

Proof. The proof is similar to the proof of Theorem 14. □

Theorem 16. *We have*

$$((F_1, A_1) \cap (F_2, A_2))^{op} = (F_1, A_1)^{op} \cap (F_2, A_2)^{op}.$$

Proof. Indeed, we have

$$\begin{aligned}
&((F_1, A_1) \cap (F_2, A_2))^{op}\\
&= (F_3, A_1 \cap A_2)^{op}, \ \text{where } F_3(e) = F_1(e) \cap F_2(e), \ \text{for } e \in A_1 \cap A_2\\
&= (F_3^{op}, A_1 \cap A_2),\\
&\qquad \text{Applying Theorem 3 we get,}\\
&= (F_3^{op}, A_1 \cap A_2),\\
&= (F_1^{op}, A_1) \cap (F_2^{op}, A_2)\\
&= (F_1, A_1)^{op} \cap (F_2, A_2)^{op}.
\end{aligned}$$

□

Theorem 17. *We have*

$$((F_1, A_1) \cap_E (F_2, A_2))^{op} = (F_1, A_1)^{op} \cap_E (F_2, A_2)^{op}.$$

Proof. The proof is similar to the proof of Theorem 16. □

Theorem 18. *We have*

$$((F_1, A_1)\tilde{\cup}_R(F_2, A_2))^{op} = (F_1, A_1)^{op}\tilde{\cup}_R(F_2, A_2)^{op}.$$

Proof. Indeed, we have

$((F_1, A_1)\tilde{\cup}_R(F_2, A_2))^{op}$
$= (F_3, A_1 \cap A_2)^{op},$ where $F_3(e) = F_1(e)\tilde{\cup}F_2(e),$ for $e \in A_1 \cap A_2$
$= (F_3^{op}, A_1 \cap A_2),$
Applying Theorem 3 we get,
$= (F_3^{op}, A_1 \cap A_2),$
$= (F_1^{op}, A_1)\tilde{\cup}_R(F_2^{op}, A_2)$
$= (F_1, A_1)^{op}\tilde{\cup}_R(F_2, A_2)^{op}.$

□

Theorem 19. *We have*

$$((F_1, A_1)\tilde{\cup}(F_2, A_2))^{op} = (F_1, A_1)^{op}\tilde{\cup}(F_2, A_2)^{op}.$$

Proof. The proof is similar to the proof of Theorem 18. □

Theorem 20. *We have*

$$((F_1, A_1) \times (F_2, A_2))^{op} = (F_1, A_1)^{op} \times (F_2, A_2)^{op}.$$

Proof. Indeed, we have

$((F_1, A_1) \times (F_2, A_2))^{op}$
$= (F_3, A_1 \times A_2)^{op},$ where $F_3((e, h)) = F_1(e) \times F_2(h),$ for $(e, h) \in A_1 \times A_2$
$= (F_3^{op}, A_1 \times A_2),$
$= (F_1^{op}, A_1) \times (F_2^{op}, A_2)$
$= (F_1, A_1)^{op} \times (F_2, A_2)^{op}.$

□

Theorem 21. *$(F_1, A_1)\tilde{\cup}_R((F_2, A_2) \cap (F_3, A_3))$ is a full soft subcategory of $((F_1, A_1)\ \tilde{\cup}_R\ (F_2, A_2)) \cap ((F_1, A_1)\tilde{\cup}_R(F_3, A_3))$.*

Proof. We have

$$\begin{aligned}
&(F_1, A_1)\tilde{\cup}_R((F_2, A_2) \cap (F_3, A_3))\\
&= (F_1, A_1)\tilde{\cup}_R(F_4, A_2 \cap A_3),\ \text{ where } F_4(e) = F_2(e) \cap F_3(e),\ \text{ for } e \in A_2 \cap A_3\\
&= (F_5, A_1 \cap A_2 \cap A_3),
\end{aligned}$$

where $F_5(e) = F_1(e)\tilde{\cup}(F_2(e) \cap F_3(e)),$ for $e \in A_1 \cap A_2 \cap A_3$.

Also, we have

$$\begin{aligned}
&((F_1, A_1)\tilde{\cup}_R(F_2, A_2)) \cap ((F_1, A_1)\tilde{\cup}_R(F_3, A_3))\\
&= (F_6, A_1 \cap A_2) \cap (F_7, A_1 \cap A_3),\\
&\qquad \text{where } F_6(e) = F_1(e)\tilde{\cup}F_2(e),\ \text{ for } e \in A_1 \cap A_2\\
&\qquad \text{and } \ F_7(e) = F_1(e)\tilde{\cup}F_3(e),\ \text{ for } e \in A_1 \cap A_3\\
&= (F_8, A_1 \cap A_2 \cap A_3),
\end{aligned}$$

where $F_8(e) = (F_1(e)\tilde{\cup}F_2(e)) \cap (F_1(e)\tilde{\cup}F_3(e)),$ for $e \in A_1 \cap A_2 \cap A_3$.

From Theorem 6, we conclude that $F_5(e)$ is a full subcategory of $F_8(e)$ for all $e \in A_1 \cap A_2 \cap A_3$. Hence, the result follows. ☐

Theorem 22. *We have*

(1) $(F_1, A_1)\tilde{\cup}((F_2, A_2) \cap (F_3, A_3))$ is a full soft subcategory of $((F_1, A_1)\ \tilde{\cup}\ (F_2, A_2)) \cap ((F_1, A_1)\tilde{\cup}(F_3, A_3))$.

(2) $(F_1, A_1)\tilde{\cup}((F_2, A_2) \cap_E (F_3, A_3))$ is a full soft subcategory of $((F_1, A_1)\tilde{\cup}(F_2, A_2)) \cap_E ((F_1, A_1)\ \tilde{\cup}\ (F_3, A_3))$.

(3) $(F_1, A_1)\tilde{\cup}_R((F_2, A_2) \cap_E (F_3, A_3))$ is a full soft subcategory of $((F_1, A_1)\tilde{\cup}_R(F_2, A_2)) \cap_E ((F_1, A_1)\tilde{\cup}_R(F_3, A_3))$.

Proof. We skip the proof as it is similar to the proof of Theorem 21. ☐

Theorem 23. *$((F_1, A_1) \cap (F_2, A_2))\tilde{\cup}_R((F_1, A_1) \cap (F_3, A_3))$ is a full soft subcategory of $(F_1, A_1) \cap ((F_2, A_2)\tilde{\cup}_R(F_3, A_3))$.*

Proof. We have

$$\begin{aligned}
&((F_1, A_1) \cap (F_2, A_2))\tilde{\cup}_R((F_1, A_1) \cap (F_3, A_3))\\
&= (F_4, A_1 \cap A_2)\tilde{\cup}_R(F_5, A_1 \cap A_3),\\
&\qquad \text{where } F_4(e) = F_1(e) \cap F_2(e),\ \text{ for } e \in A_1 \cap A_2\\
&\qquad \text{and } \ F_5(e) = F_1(e) \cap F_3(e),\ \text{ for } e \in A_1 \cap A_3\\
&= (F_6, A_1 \cap A_2 \cap A_3),\\
&\qquad \text{where } F_6(e) = (F_1(e) \cap F_2(e))\tilde{\cup}(F_1(e) \cap F_3(e)),\ \text{ for } e \in A_1 \cap A_2 \cap A_3.
\end{aligned}$$

Also, we have

$$\begin{aligned}&(F_1, A_1) \cap ((F_2, A_2)\tilde{\cup}_R(F_3, A_3))\\&= (F_1, A_1) \cap (F_7, A_2 \cap A_3),\ \text{where } F_4(e) = F_7(e)\tilde{\cup}F_3(e),\ \text{for } e \in A_2 \cap A_3\\&= (F_8, A_1 \cap A_2 \cap A_3),\end{aligned}$$

where $F_8(e) = F_1(e) \cap (F_2(e)\tilde{\cup}F_3(e))$, for $e \in A_1 \cap A_2 \cap A_3$.

From Theorem 7, we conclude that $F_6(e)$ is a full subcategory of $F_8(e)$ for all $e \in A_1 \cap A_2 \cap A_3$. Hence, the result follows. ☐

Theorem 24. *We have*

(1) $((F_1, A_1) \cap (F_2, A_2))\tilde{\cup}((F_1, A_1) \cap (F_3, A_3))$ *is a full soft subcategory of* $(F_1, A_1) \cap ((F_2, A_2)\tilde{\cup}(F_3, A_3))$.

(2) $((F_1, A_1) \cap_E (F_2, A_2))\tilde{\cup}((F_1, A_1) \cap_E (F_3, A_3))$ *is a full soft subcategory of* $(F_1, A_1) \cap_E ((F_2, A_2)\tilde{\cup}(F_3, A_3))$.

(3) $((F_1, A_1) \cap_E (F_2, A_2))\tilde{\cup}_R((F_1, A_1) \cap_E (F_3, A_3))$ *is a full soft subcategory of* $(F_1, A_1) \cap_E ((F_2, A_2)\tilde{\cup}_R(F_3, A_3))$.

Proof. We skip the proof since it is similar to the proof of Theorem 23. ☐

Theorem 25. *We have*

$$((F_1, A_1)\ AND\ (F_2, A_2))^{op} = (F_1, A_1)^{op}\ AND\ (F_2, A_2)^{op}.$$

Proof. Indeed, we have

$$\begin{aligned}&((F_1, A_1)\ AND\ (F_2, A_2))^{op}\\&= (F_3, A_1 \times A_2)^{op},\ \text{where } F_3((e, h)) = F_1(e) \cap F_2(h),\ \text{for } (e, h) \in A_1 \times A_2\\&= (F_3^{op}, A_1 \times A_2),\\&= (F_1^{op}, A_1)\ AND\ (F_2^{op}, A_2)\\&= (F_1, A_1)^{op}\ AND\ (F_2, A_2)^{op}.\end{aligned}$$

☐

Theorem 26. *We have*

$$((F_1, A_1)\ OR\ (F_2, A_2))^{op} = (F_1, A_1)^{op}\ OR\ (F_2, A_2)^{op}.$$

Proof. We have

$$\begin{aligned}
&((F_1, A_1)\ OR\ (F_2, A_2))^{op} \\
&= (F_3, A_1 \times A_2)^{op}, \text{ where } F_3((e,h)) = F_1(e) \cup F_2(h), \text{ for } (e,h) \in A_1 \times A_2 \\
&= (F_3^{op}, A_1 \times A_2), \\
&= (F_1^{op}, A_1)\ OR\ (F_2^{op}, A_2) \\
&= (F_1, A_1)^{op}\ OR\ (F_2, A_2)^{op}.
\end{aligned}$$

□

Note that the operations union, restricted union, intersection, extended intersection, AND, OR in soft category are just the generalizations of union, restricted union, restricted intersection, extended intersection, AND, OR in soft set respectively. So the theorems above on these operations are also generalization of the corresponding theorems of soft set.

5 Composition of soft functors

In this section, we introduce the notion of composition of soft functors and form the category of all soft categories.

Let (F_1, A_1), (F_2, A_2) and (F_3, A_3) are soft categories over the categories C_1, C_2 and C_3 respectively. Let (K_1, g_1) and (K_2, g_2) be soft functors from (F_1, A_1) to (F_2, A_2) and (F_2, A_2) to (F_3, A_3), respectively. Then, (K, g) is said to be the *composition of these soft functors* and defined to be $(K_2 \circ K_1, g_2 \circ g_1)$.

Now, we show that (K, g) is a soft functor from (F_1, A_1) to (F_3, A_3). First of all we observe that, being composition of two full functors, K is a full soft functor from C_1 to C_3. Secondly, it is clear from the context that g is a surjection from A_1 to A_3. And last but not the least, we see that,

$$\begin{aligned}
K(F_1(x)) &= (K_2 \circ K_1)(F_1(x)) \\
&= K_2(K_1(F_1(x))) \\
&= K_2(F_2(g_1(x))), \text{ as } (K_1, g_1) \text{ is a soft functor,} \\
&= F_3(g_2(g_1(x))), \text{ as } (K_2, g_2) \text{ is a soft functor,} \\
&= F_3((g_2 \circ g_1)(x)) \\
&= F_3(g(x)).
\end{aligned}$$

Hence, the composition of two soft functors is again a soft functor.

Furthermore, we observe that, for each soft category (F, A) over C there exists an ‘identity’ soft functor, namely (I_C, i_A), where I_C is the identity functor on the category C and i_A is the identity function on the set A, in the sense that given any soft category (G, B) over D and a soft functor (K, g) from (F, A) to (G, B) or from (G, B) to (F, A), $(I_C, i_A) \circ (K, g) = (K, g)$ or $(K, g) \circ (I_C, i_A) = (K, g)$, respectively.

Now, we are going to prove that, associativity holds for composition of soft functors. Let (F_1, A_1), (F_2, A_2), (F_3, A_3) and (F_4, A_4) are soft categories over the categories C_1, C_2, C_3 and C_4, respectively. Let (K_1, g_1), (K_2, g_2) and (K_3, g_3) be soft functors from (F_1, A_1) to (F_2, A_2), (F_2, A_2) to (F_3, A_3) and (F_3, A_3) to (F_4, A_4), respectively. Then,

$$\begin{aligned}
((K_3, g_3) \circ (K_2, g_2)) \circ (K_1, g_1) &= (K_3 \circ K_2, g_3 \circ g_2) \circ (K_1, g_1) \\
&= ((K_3 \circ K_2) \circ K_1, (g_3 \circ g_2) \circ g_1) \\
&= (K_3 \circ (K_2 \circ K_1), g_3 \circ (g_2 \circ g_1)) \\
&= (K_3, g_3) \circ (K_2 \circ K_1, g_2 \circ g_1) \\
&= (K_3, g_3) \circ ((K_2, g_2) \circ (K_1, g_1)).
\end{aligned}$$

All the results, we proved above, implies that the class of all soft categories along with the soft functors form a category which we denote by $SCAT$. It is also worthy to note that, for a given category C, all soft categories over C is a full subcategory of $SCAT$ which we denote by $C - SCAT$.

6 Conclusion

Both the category theory and soft set theory play vital roles in several areas like engineering, medical sciences, supply chain management etc. Category theory is ideal for reasoning about structure, abstracting away from details, and automation. Many branches like type theory, programming language semantics, topos theory etc have strong categorical theoretical background. On the other hand, soft set theory, as a tool of soft computing, individually or in integrated manner, is turning out to be a strong candidate for performing tasks in the area of data mining, decision support systems, supply chain management, medicine, data compression etc. So, in the light of this paper, one can find some useful application using this new algebraic structure of soft category. Also, one can try to define more operations in soft category and find relationship between them.

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